Magnetic variogram depth: tests on synthetic and real data

S. Maus, TU Braunschweig, and B. Röttger and K.-P. Sengpiel, BGR Hannover*

Summary

We investigate the accuracy and resolution of the variogram analysis method (Maus, 1999) on synthetic and real data. Synthetic magnetic flight line data are generated for basement models of idealized geological setups. Comparing variogram depths of the data with true model depths shows that variogram depths are accurate and unbiased as long as the data analysis window is larger than 10 times the depth to be estimated. However, basement features smaller than this window size are not resolved. We investigate the trade-off between window size and lateral resolution by comparing variogram depth with drilled basement depth in the Kuiseb Dune Area, Namibia. For this difficult data set the optimum window size is more than 20 times the depth to be resolved. A provocative conclusion from our study is that one should consider flying survey lines in a regular mesh. This optimizes the retrieval of depth information per line km.

Introduction

Depth can be estimated from magnetic data by inverting individual anomalies, e.g. Euler deconvolution (Reid et al., 1990), or by statistical methods which make use of spectral properties of the field. Spector and Grant's original method (1970) can be improved by taking into account the self-similar (fractal) nature of source (Pilkington et al., 1994; Maus and Dimri, 1996). Directly analyzing line data using variograms (Maus, 1999; Maus et al., 1999) leads to further significant improvements in accuracy and resolution. The aim of the present study is to verify the quality of variogram depth on synthetic and real data.

We generate a self-similar magnetized basement and synthesize magnetic data in flight lines above this model (Fig. 1). Resolution of alternating slopes and channels depends on the relative size of the data analysis window. Features larger than this window size are generally well resolved. Variogram depth is correct on average, hence, it is unbiased. However, depth estimates can over- and undershoot at topographic gradients. Basement ruggedness, unknown scaling exponent of source and non-negligible sediment magnetization may cause further problems, which are studied here. An interesting question is, whether it is preferable to fly along or across the basement topographic trend. The best strategy is to do both, namely, to fly with twice the line spacing in both directions. This regular mesh offers the optimum basement resolution per line km surveyed.

Finally, we process a magnetic survey of the Kuiseb Dune Area. Interpretation is complicated by high instrument noise and variable surface topography. Comparing basement reliefs with drilling results in 15 locations we find an optimum data analysis window size of 5400 m for this survey. This is disappointingly large. Arguably, resolution would have been better if survey lines had followed the dune valleys.

Synthetic modeling

We generate realistic magnetic flight line data from self-similar source models with imposed topography. Depth estimated from this data is compared with true model depths (e.g. Fig. 1) to test the variogram analysis method.

2D equivalent layer models. Any 3d magnetization model can be replaced by an equivalent layer of induced magnetization at the model surface, drastically reducing computation time. A self-similar 3d susceptibility distribution with scaling exponent β causes a magnetic field with scaling exponent $\beta - 1$ in a horizontal observation plane (Maus and Dimri, 1996). The same magnetic field is caused by an equivalent 2d susceptibility layer with scaling exponent $\beta + 1$. Hence, we can substitute a 3d source model with scaling exponent β by a 2d layer with $\beta + 1$.

Self-similar grid synthesis. We fill a grid with random numbers in the wavenumber domain in such a way that the corresponding grid in the space domain has the desired spectrum (Maus and Dimri, 1996). With this method grids can be synthesized which are self-similar up to lags of around 1/4th of the grid size. In the following experiments the data analysis window size is always within the self-similar range.

Synthetic magnetic data. Once a self-similar grid is synthesized, the desired models can be produced by superimposing topography. This is valid if vertical variations are small compared with the horizontal scale of the equivalent layer. We use a long strip of basement. Topography is 1d in the sense that it varys along the strip and is constant perpendicular to the strip. Two data sets are synthesized for every model. One with flight lines parallel to the strip (across topography) and one with lines perpendicular to the strip (Fig. 1). Magnetic ΔT-data are computed for a vertical inducing field. In the models, every grid cell represents 10 m x 10 m in nature. However, all model dimensions and depths may be scaled by a constant factor.

Variogram analysis algorithm. We use the same algorithms as in earlier work (Maus, 1999; Maus et al., 1999). The only new feature is that when we estimate depth in a particular location, we average variograms weighted by a Gauss bell shaped function $\exp(-r^2/\sigma^2)$. At $r = \sigma$

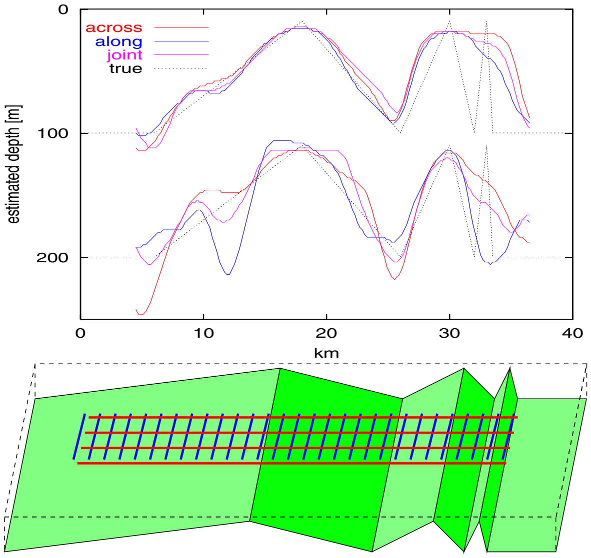

Fig. 1: Basement topography is superimposed on a 2d equivalent layer with a self-similar distribution of susceptibility. Depth is estimated from synthetic magnetic data computed for flight lines in horizontal observation planes above the basement model. Note that the vertical scale of the model is exaggerated.

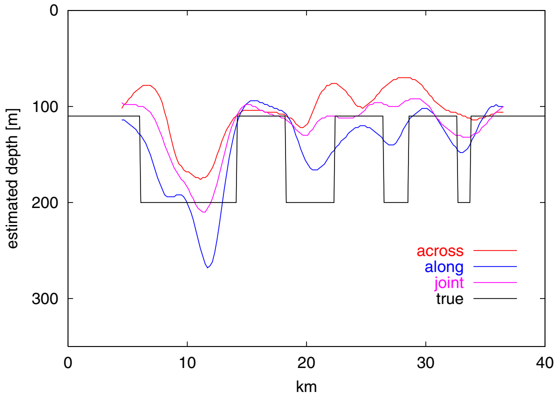

Fig. 2: Depth to a basement with channels

the weight function falls off to $e^{-1} \approx 0.37$. We refer to the diameter 2σ as the effective window size. Contributions from further than $r = 3\sigma$, where the weight drops to below 10^{-4}, can be ignored. Using this weight function does not improve the accuracy of depth, but leads to a reasonably smooth basement relief. With uniform weights, instead, the relief shows circular artifacts reflecting the cut-off radius. In the synthetic modeling we use an effective window size of 4 km and estimate depth from profiles oriented *across* as well as *along* topography. We also estimate a *joint* depth, where we simultaneously invert half of the profiles in both directions.

Sloping basement. Topography with alternating gradients of increasing slope is superimposed onto the basement strip. Estimated depth is shown in Fig. 1. The first three slopes are mapped rather accurately. The length of the third slope is 4 km and thus equal to the effective window size used. Features smaller than this window size are not resolved. Depths from the across topography profiles tend to overshoot. The reason is that when a profile runs from deep basement into shallow basement, a topographic magnetic anomaly ensues. This long wavelength anomaly increases the ratio of long to short wavelength power which is interpreted as greater depth to source. Thus, topographic anomalies can lead to overshooting depth estimates. Along-topography depth does not show this artifact and the most accurate depth is obtained from the bi-directional inversion.

Paleochannels. In locating paleochannels, we would like to know how broad a channel must be in order to be identified by the variogram analysis. We indent channels of decreasing width into the basement strip (Fig. 2) and try to locate them using an effective 4 km x 4 km variogram analysis window size. We find that a channel has to be roughly as broad as the size of the analysis window in order to be identified reliably.

Unknown scaling exponent. In practice, the scaling exponent of the basement magnetization can only be guessed. We estimate depth from profiles at 10 m to 600 m height above a flat basement model and find that a 0.5 uncertainty in the scaling exponent leads to an uncertainty of around 20% in absolute depth. This uncertainty of depth versus smoothness of source follows from the non-uniqueness of the magnetic inverse problem.

Rugged basement. Small scale variations in basement topography cannot be resolved by variogram analysis. Instead, they lead to a bias in basement depth estimates. A source grid is combined with a self-similar topography grid, representing a basement with self-similar magnetization as well as topography. The superimposed topography grids with $\beta = 2$ are chosen to have zero mean value, 10 m, 20 m, and 50 m standard deviation, and maximum values of 50 m, 100 m, and 250 m, respectively. We find that a rugged basement leads to reduced depth estimates (Fig. 3). As a rule of thumb, the estimated depth lies about half-way between the mean and the minimum depth to basement.

Non-negligible sediment magnetization. High resolution magnetic surveys are increasingly employed to map intra-sediment anomalies. Does this contradict the basic assumption of a negligible sediment magnetization in the variogram analysis? We investigate how strong the magnetization of sediments may be without affecting depth estimates. Assuming a survey terrain clearance of 50 m, sediment magnetic fields of decreasing strength are added to the magnetic fields of basements at depths of 100 m, 200 m and 300 m, respectively. Estimated depth is plotted against the ratio of basement to sediment magnetization in Fig. 4. The result is reassuring. Even for the greatest depth, corresponding to one tenth of the window size, a susceptibility contrast of around two orders of magnitude is sufficient to allow reliable depth estimation. This is not a very strong contrast for a typical basin.

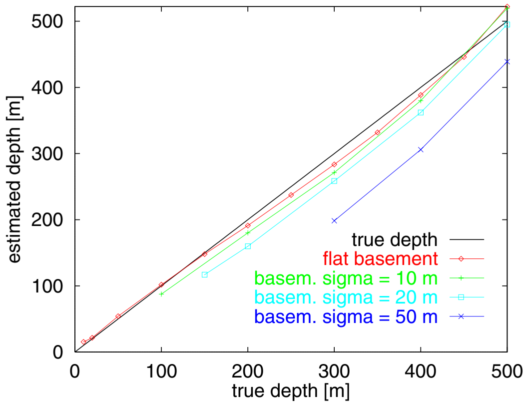

Fig. 3: Estimated versus true depth for rugged basement

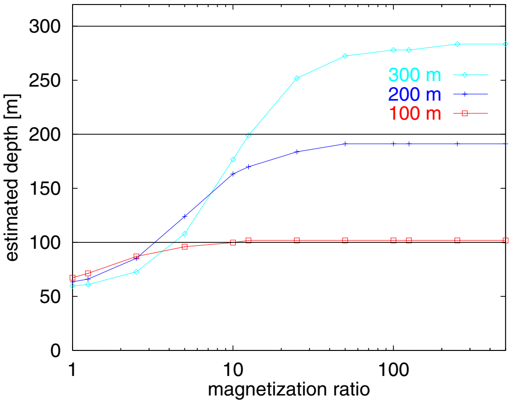

Fig. 4: Depth against basement to sediment magnetization

Helicopter survey of the Kuiseb Dune Area

In 1992, the Kuiseb Dune Area in Namibia was surveyed by helicopter for its ground water potential. 12000 line-km were flown with a line spacing of 0.4 to 1 km and a tie line spacing of 5 to 10 km (Fig. 5). It was known that part of the water of the Kuiseb River seeps into the dune area and re-emerges on the coast feeding freshwater lagoons. Aim of the survey was to identify these seepage paths beneath the sand dunes. Indeed, EM resistivities indicated some high resistivity fresh water channels. Subsequent drilling confirmed these channels, but fresh water yields were insignificant.

Variogram analysis of the magnetic data set faces difficulties. Sand dunes of 100 m in height are criss-crossed by the draped survey lines. Upward continuation to a common reference height is not possible for profile data without making assumptions on the source magnetization. Thus, our observation "plane" actually consists of measurements vertically separated by as much as 100 m, while the depth to be estimated is of the order of 150 m! The presence of instrument noise of up to several nT causes further problems.

To obtain meaningful depth, large variogram window sizes have to be used. The larger the window, the more accurate the depth estimate. However, this increased accuracy trades off against a decreased lateral resolution of the basement relief. We infer the mean error of the relief from drilled basement depth in 15 locations. Plotting the mean error against the effective window size, we find that accuracy of depth improves up to an optimum window size of 5400 m, where the mean relative error is 15% and the absolute error is 25 m. For even larger windows, the increasing accuracy is offset by the decreasing lateral resolution. Fig. 5 shows the basement relief for a window size of 4500 m. The large window size required is primarily due to the varying flight altitude above basement. One could try to upward continue the profiles to a reference plane sloping towards the sea. However, this requires assumptions on the source magnetization. In an alternative approach, we discard all data measured above the mean local observation height, shedding 50% of the data. Due to the chopped up profiles, 3/4 of the variograms become incomplete and must be discarded. Consequently, instead of improving, depth estimates deteriorate. Attempts to incorporate the tie lines in a joint inversion also fail to improve the relief. For this purpose the tie line separation would have had to be narrower.

Discussion and Conclusions

Our tests show that variogram analysis depth is unbiased and accurate, subject to certain limitations. Some of these limitations are common to other methods as well.

Synthetic modeling. Comparing variogram depth with true model depth we conclude that

1. the effective data analysis window size must be at least 10-20 times larger than the depth to be resolved
2. topographic features are reliably identified only if they are larger than the window size (Figs. 1, 2)
3. for rugged basement topography, estimated depth is about half-way between the mean and the minimum depth to basement (Fig. 3)
4. due to the inherent non-uniqueness of the magnetic inverse problem, a 0.5 uncertainty in β (which is rather high) leads to a 20% uncertainty in depth
5. a sediment magnetization up to two orders of magnitude less than the basement magnetization does not disturb depth estimates (Fig. 4)
6. depth estimated from profiles along constant basement depth is more accurate than from profiles across topography. Hence, flight lines should be aligned with topographic trend to avoid topographic anomalies. Tie lines should be flown dense enough to be useful as an additional source of statistical information. In particular, joint inversion of wide spaced bi-directional profiles gives more accurate depth than a dense equi-directional set of survey lines (Figs. 1, 2).

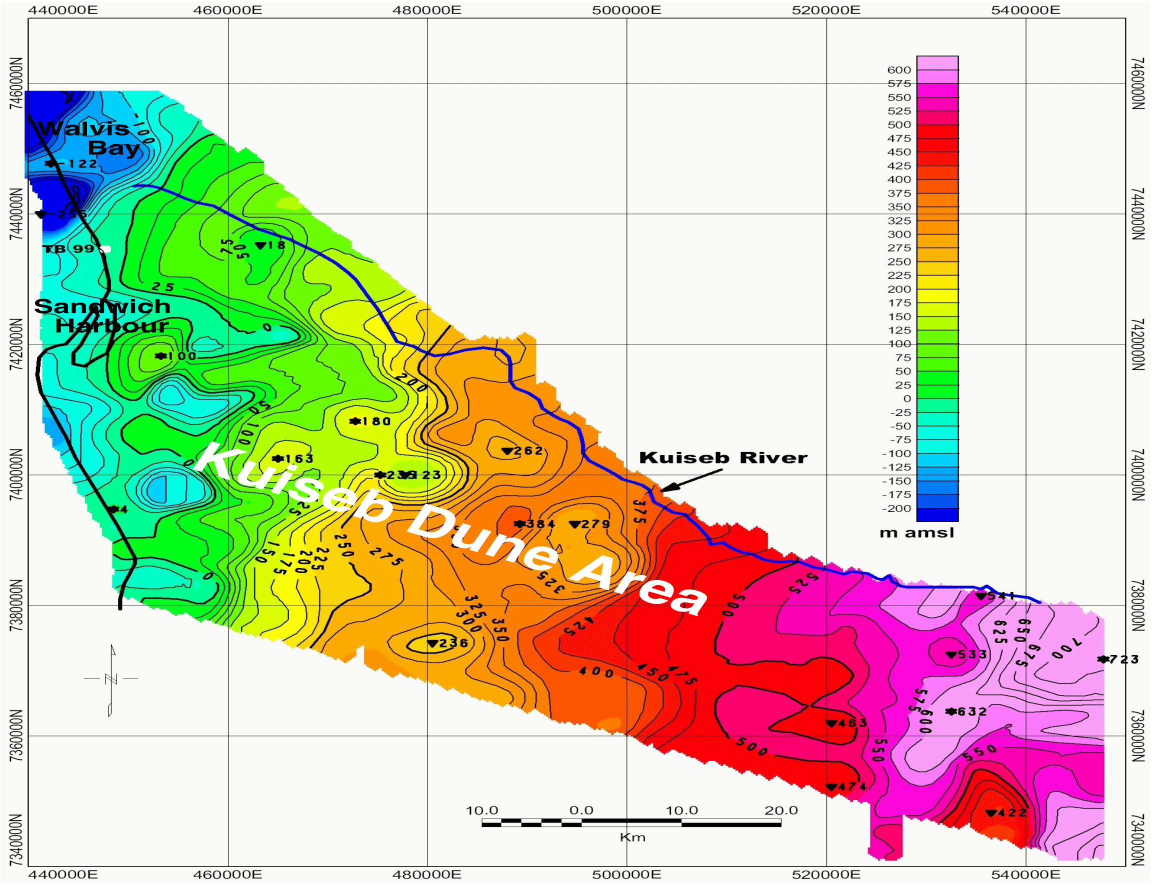
Magnetic variogram depth

Fig. 5: Basement relief estimated using a variogram analysis window size of 4500 m. The relief for the optimum window size of 5400 m has smaller deviations from drilled depth, but shows even less detail.

Kuiseb Dune Area survey . Variable dune topography and high instrument noise make it difficult to estimate the basement relief from the Kuiseb Dune Area magnetic data. The absolute error of around 25 m against drilled depth is likely to be even larger at the survey boarders, where the analysis window is only half covered with data. While basement depths of 100 m found for Walvis Bay are realistic, the strong gradient to even greater depths in the West is not. The fresh water channel feeding the wetland at Sandwich Harbour has been identified correctly. Presumed troughs along the coast roughly agree with areas of high EM resistivity. It would be interesting to verify the existence of these troughs by drilling.

Most likely, depth estimates would have been more accurate if survey lines had followed the dune valleys and more tie lines had been flown.

Aeromagnetic survey design. Our synthetic modeling shows that the information in tie lines could significantly improve depth estimates, provided that the tie lines are not spaced too far apart. In fact, for basement depth estimation, flying a regular mesh would probably give the optimum accuracy at least cost.

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