

FORUM

Comment on "Error Made in Reports of Main Field Decay"

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As the International Association of Geomagnetism and Aeronomy (IAGA) Working Group on Geomagnetic Field Modeling (<http://www.ngdc.noaa.gov/IAGA/vmod/>), responsible for the International Geomagnetic Reference Field (IGRF) [Macmillan *et al.*, 2003], we would like to comment on the Forum article by Wallace H. Campbell (*Eos*, 85(16), 20 April 2004). Campbell claims that reports of dipole decay at a special session held at the AGU 2003 Fall Meeting were misleading due to an incorrect choice of the coordinate system for the spherical harmonic analysis (SHA) of the geomagnetic field used for the IGRF; the model on which the decay calculation was based.

Campbell alleges that the dipole moment of a spherical harmonic expansion depends on the choice of the origin of the coordinate system. In his textbook on geomagnetism, Campbell goes one step further in asserting that, without changing the origin, the process of "tilting the analysis axis to align with the geomagnetic axis... would enhance the dipole term at the expense of the higher multipoles" [Campbell, 2003]. Remembering that (because of the absence of magnetic monopoles) the magnetic dipole vector moment is physically invariant, and also some basic properties about orthogonal functions, many readers must have been puzzled by these statements. In fact, these statements are incorrect, as the dipole coefficients (G_1^0 , G_1^{-1} , and H_1^{-1}) are invariant under displacements of the origin [see, e.g., Langel, 1987] and the dipole moment is independent of the orientation of the coordinate system.

Campbell also implies that there are correct and incorrect coordinate systems when he states in the Forum article, "Only one SHA provides the correct dipole coefficients for determining the 'Decay of the Main Field'—the one

centered on the eccentric dipole location that is determined by a computational process of choosing the best analysis center that minimizes the higher-than-dipole coefficients." Apart from the invariance of the vector dipole moment, coordinate systems are usually chosen to best reflect the symmetries of the physical system. Because of the symmetry of the core, and the influence of Earth rotation on fluid flow, a natural choice of the spherical coordinate system in geomagnetism is a geocentric one, with the $\theta=0$ axis aligned with the spin axis. In order to make a case for the proposed dipole-centered, eccentric coordinate system, one has to attribute a physical significance to the magnetic dipole itself. Indeed, Campbell writes that "These (higher multiple) coefficient values are large mainly because of the physical offset of the eccentrically located Earth's dipole field." However, an explicit dipole source moving around in the Earth's core is not a physical concept generally endorsed by the geomagnetic community. Indeed, in his paper "The geomagnetic eccentric dipole: Facts and fallacies," Lowes [1994] explicitly warns about attempts to give a physical significance to the position of the eccentric dipole. The reason that the dipole moment is often treated as special is that it is the only simple invariant property of the geomagnetic field; it is also a useful first approximation numerically, as, for most of the time, at the Earth's surface it does in fact dominate the field that we observe.

In claiming that the choice of an Earth-centered SHA for the IGRF is arbitrary and motivated by computational convenience, Campbell misses a further important point. While an Earth-centered coordinate system is indeed convenient, it is also logical. The SHA has to be Earth-centered because a scalar magnetic potential is valid only outside of the source region. Since the IGRF represents the

entire long-wavelength internal field, including inseparable contributions from the lithospheric magnetization, a SHA center offset by 500 km could only be used to represent the field at distances greater than about R_e+500 km from this center. Also, even if the position of the "eccentric dipole" had physical significance (which we dispute), this position is only known approximately; there are a variety of ways of determining it, and it is changing with time. Users who wish to use off-centered dipole coordinates can always evaluate the IGRF in geocentric coordinates and then transform the field values to their preferred system.

In summary, there is no credible alternative to the geocentric coordinate system used for the IGRF and other main field models. In particular, the IGRF provides the correct and only vector dipole moment of the geomagnetic field, within reasonable error bounds. In his final paragraph, Campbell reminds us that "AGU scientists have an obligation to report to the public in their best representations of our environment, not values distorted by arbitrary selections in the analyzing technique." Indeed, the IAGA Working Group on Geomagnetic Field Modeling supports fair and accurate reporting and hopes that this response clears up any confusion resulting from Campbell's Forum article.

References

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- Langel, R.A. (1987), The main geomagnetic field, in *Geomagnetism*, vol. 1, edited by J.A. Jacobs, chap. 4, pp. 249–512, Academic, San Diego, Calif.
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—STEFAN MAUS, NOAA National Geophysical Data Center, Boulder, Colo.; SUSAN MACMILLAN, British Geological Survey, Edinburgh, U.K.; and other members of the IAGA Working Group VMOD on Geomagnetic Field Modeling

Reply

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I thank Maus et al. for providing this opportunity to explain spherical harmonic analysis (SHA) methods to our *Eos* readers. Gauss devised the SHA as a means for separating the external and internal geomagnetic field contributions at an analysis spherical surface. The SHA starts with the selection of an analysis center and axis through that center defining the study coordinates of longitude Φ , latitude θ , and radius r of the analysis sphere. Using Maxwell's equations, the three orthogonal geomagnetic field components at the analysis spherical surface are converted to equivalent potential function values to which two special series of terms are then fitted.

One series has coefficients of increasing powers of r (the external separated field part)

and the other has coefficients of increasing powers of $1/r$ (the internal part). The sought internal field is then represented by a double summation of "order" m ($m=0$ to n) and "degree" n ($n=1$ to ∞ , with $n \geq m$) containing coefficients g_n^m and h_n^m for the cosine($m\Phi$), sine($m\Phi$), and Legendre polynomials, $P_n^m(\theta)$. This procedure is somewhat like fitting the daily amplitude of quiet daily field change to a Fourier series of the harmonic oscillating ups and downs of the cosine and sine terms; only now we are fitting the surface variations of the potential function to the appropriate amplitudes of the bulges and depressions in the harmonic polynomials over a spherical surface, with their symmetry defined by Φ and θ .

To appreciate the SHA dependence upon the selection of the Φ and θ coordinates, and to see how this selection affects the fitting harmonic composition, the reader should examine

the illustrations of P.McFadden (Demonstration of spherical harmonics, 1995; go to <http://www.ngdc.noaa.gov/seg/potfld/geomag.shtml>; download and run program SPH). The SHA polynomials have $n-m+1$ Legendre (sinusoidal-like) waves (or just n waves if $m=0$) fitted around a great circle of longitude Φ , and m sinusoidal waves around each circle of latitude θ . Symmetry about the longitude and latitude for these SHA polynomials is fixed by the selection of the analysis sphere Φ and θ . Special groupings of the SHA harmonics can be added together to produce potential functions resembling the displays expected from a dipole electric field (opposing charges at ends of an axial line segment), a quadrupole field (opposing charges at corners of a square arrangement), an octupole field (opposing charges at the corners of a cube), etc., all centered symmetrically about the chosen

analysis sphere. Although a change in tilt of the analysis axis will result in changes of the SHA coefficients, the polynomial groupings remain the same for an unchanged axis center.

For convenience, the IGRF uses the Earth's center and spin axis for definition of a Φ and θ coordinate system and has the analysis spherical surface located at an r just above the Earth's surface. Using a summation of all the IGRF SHA coefficient terms (above noise levels), one can properly represent the Earth's main field internal to the analysis sphere. A change in the analysis center would alter the analysis Φ and θ with respect to the fixed magnetic field potential function, resulting in a significant change in the g_n^m and h_n^m coefficient amplitudes within the polynomial groupings for the SHA fitting. The summation of all these new polynomials would also fully represent the Earth's main internal field; the IGRF coefficients are not unique.

If the Earth's field were just a simple dipole located at the Earth's center and aligned along, or tilted to, the spin axis, then only a set of SHA dipole harmonics in Earth-centered coordinates would be needed for representation. However, charts of the Earth's main field, the observed asymmetric location of the magnetic pole positions, the L-shell contours (defined by trapped particle flux), the auroral oval, the South Atlantic field anomaly (with its attending satellite damage), the true dip-equator offset from the geomagnetic (IGRF dipole) equator, etc., all indicate that the true dipole part of the Earth's main field has a center that is significantly offset from the Earth's center and tilted with respect to the Earth's spin axis. To find the best analysis center and dipole tilt, scientists search for a new analysis Φ and θ that will maximize the SHA dipole components at the expense of minimizing the other SHA harmonic components. That dipole at the new

location is called the "eccentric axis dipole." Because the quadrupole component of the IGRF is the largest of the multipolar groups, it is often considered sufficient, in the search, to just minimize those quadrupole terms.

The arguments of Maus et al. for an Earth-centered main field are curiously reminiscent of the dogmatic proclamations by the early church favoring an Earth-centered solar system (citing authorities Aristotle and St. Thomas Aquinas), despite proof to the contrary by Copernicus and Galileo. It is extremely fortunate for me that the august IAGA group does not have the enforcement powers of the seventeenth-century Roman Inquisition.

—WALLACE H. CAMPBELL, Boulder, Colo.

Campbell will be leaving his Guest Scientist position at the Solar-Terrestrial Physics Division of NGDC/NOAA about the end of September 2004.

ABOUT AGU

Outstanding Student Paper Awards: Ocean Sciences; Space Physics and Aeronomy; Volcanology, Geochemistry, and Petrology

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The following members received Outstanding Student Paper Awards at the 2004 AGU/CGU Joint Assembly in Montreal, Canada. (Winners in other sections and focus groups will be announced in subsequent issues of Eos.)

Ocean Sciences

Alexander Korobov, University of Waterloo, *Energy transfer in internal waves generated by tidal flow over topography*

Gregory Smith, McGill University, *On the formation and circulation of the intermediate waters of the Gulf of St. Lawrence*

Denis Volkov, Royal Netherlands Institute for Sea Research, *The variability of sea level in*

the North Atlantic and North Pacific in 1993–2002 observed with satellite altimetry

Space Physics and Aeronomy

David Berube, University of California, Los Angeles, *Plasmapheric mass density response to geomagnetic storms determined from ULF resonance data*

Christy Bredeson, University of Alberta, *Field testing and calibration of low-cost pulse-counting magnetometers*

Anna D. DeJong, University of Michigan, Ann Arbor, *Steady magnetospheric convection during various levels of magnetic activity*

Jared R. Espley, Rice University, *Low altitude low frequency magnetic oscillations at Mars*

Chia-Lin Huang, Boston University, *Skill score analysis of MHD simulation global magnetic fields*

Michele Kuester, Colorado Research Associates, *Observed and modeled stratospheric gravity waves above Hurricane Humberto*

Daniel S. Main, University of Colorado, Boulder, *Modeling the lower boundary of the upward current region as an oblique double layer: effects on the evolution of O⁺ density of ionospheric origin*

Michael J. Nicolls, Cornell University, *Observations of a large-scale thermospheric gravity wave using ISR, TEC, and airglow data and its effect on the mesosphere*

Marila Samara, Dartmouth University, *Analysis of ionospheric waves observed with the SIERRA sounding rocket*

Volcanology, Geochemistry, and Petrology

Philip Benson, University College London, *Modelling the physical properties of cracked rocks (2): application to transversely isotropic granite from the Japanese Alps*

Conor Gately, Wesleyan University, *Geochemistry of the hydrothermal system of Peteroa Volcano, Chile-Argentina*