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RESEARCH NOTE

Is there an observable lack of reciprocity in *PKP*(*DF*) traveltimes?

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SUMMARY

On average, traveltimes of PKP_{DF} for equatorial ray paths through the quasi-eastern hemisphere of the inner core are around 0.5 s faster than equivalent ray paths through its quasi-western hemisphere. In these observations, the eastern hemisphere is sampled primarily by westward- and the western hemisphere by eastward-propagating waves. Noting that westward propagation is faster than eastward propagation inside a rotating earth, I estimate the expected traveltime difference from Coriolis splitting of the displacement eigenfunctions of the PKP_{DF}-equivalent modes. It turns out that Coriolis effects are too small to give rise to residuals of the required magnitude. Thus, the observations must be primarily due to velocity heterogeneities.

Key words: inner core anisotropy, *PKP*(*DF*), seismic traveltimes.

1 INTRODUCTION

Analysing the traveltimes of seismic core phases from bulletins of the International Seismological Centre (ISC), Poupinet *et al.* (1983) found that seismic waves travel faster through the Earth's inner core on polar than on equatorial paths, even after correcting for ellipticity. These anomalous traveltime residuals are particularly obvious in differential core phase observations (Cormier & Choy 1986; Creager 1992). Core anisotropy is also apparent in free oscillations of the Earth, where core-sensitive modes are more strongly split than predicted for a rotating isotropic earth in hydrostatic equilibrium (Masters & Gilbert 1981). A detailed review of inner core anisotropy is given by Song (1997).

In addition to anisotropy, differential seismic traveltimes indicate large-scale heterogeneity in the inner core (Creager 1992). *P* velocities seem faster in the quasi-eastern hemisphere and slower in the quasi-western hemisphere of the inner core (Tanaka & Hamaguchi 1997; Creager 1999). Interestingly, the eastern hemisphere is sampled primarily by westward- and the western hemisphere by eastward-propagating waves. Hence, the phenomenon could also be explained by a general lack of reciprocity, with faster westward and slower eastward propagation. The most likely cause of such a lack of reciprocity is the Coriolis force of Earth rotation, which accelerates westward (retrograde) and decelerates eastward (prograde) propagation.

After briefly illustrating the residuals observed in absolute and differential traveltimes, I derive below a quantitative estimate of the Coriolis effect on PKP_{DF}. The expected traveltime difference between eastward and westward propagation turns out to be of the order of 10^{-4} s, about three orders of magnitude smaller than the average residuals to be explained.

2 OBSERVED TRAVELTIMES

While absolute traveltime observations of PKP_{DF} are available in large numbers, hand-picked differential PKP_{AB}–PKP_{DF} and PKP_{BC}–PKP_{DF} traveltimes have the advantage of lower noise and less contamination by event mislocations and velocity heterogeneities in the crust and mantle. Here, I display both kinds of data.

To avoid the known effects of inner core anisotropy, I discard polar ray paths with ray angles at a turning point of less than 40° against the Earth's spin axis. Observations are classified into two groups, one with eastward (prograde) and one with westward (retrograde) propagation. Finally, residuals are displayed against the ray turning point longitude, averaged in 30° bins.

2.1 Absolute PKP_{DF} traveltimes

These data are taken from reprocessed ISC Bulletin events (Engdahl *et al.* 1998). I use only arrivals from epicentral distances greater than 148° in order to avoid misclassifications around the 145° range, where traveltimes of PKP_{AB}, PKP_{BC} and PKP_{DF} are nearly equal. After applying elevation, station and ellipticity corrections, I compute the residuals against

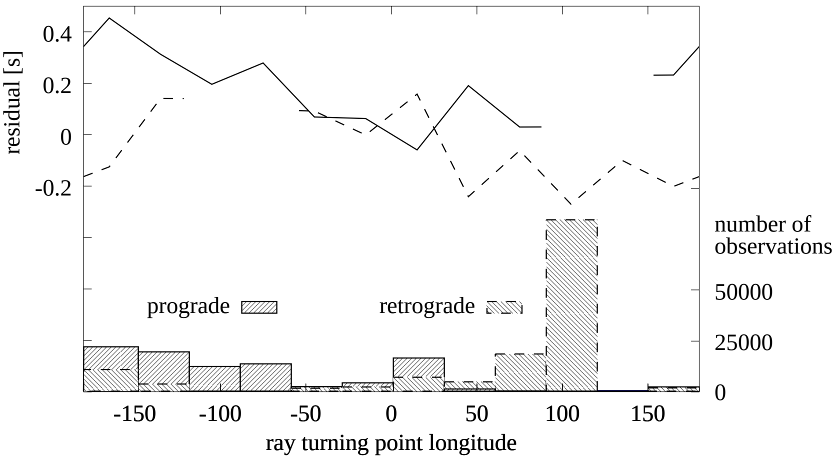

Figure 1. ISC traveltime residuals of PKP_{DF} for non-polar paths, averaged in 30° turning-point longitude intervals. Average residuals of all intervals with more than 250 observations are displayed as prograde and retrograde curves. The histograms illustrate the difference in longitudinal distribution of observations between the two groups.

AK135 (Kennet & Engdahl 1995) and reject values greater than 5 s. The resulting data set of 228 011 residuals has a standard deviation of 1.46 s.

2.2 Differential traveltimes

Differential PKP_{BC}–PKP_{DF} traveltimes were hand-picked by Kenneth Creager from 2500 short- and intermediate-period recordings of the Global Digital Seismograph Network (GDSN) from 1980 to 1986 (Creager 1992). After excluding polar ray paths, the standard deviation of the remaining 244 residuals to AK135 is 0.45 s.

The differential PKP_{AB}–PKP_{DF} data set of McSweeney *et al.* (1997) was compiled by a cross-correlation method from digital vertical-component short-period seismograms of the GDSN, the Alaska Seismic Network (ASN) and the CALNET regional array in northern California. After excluding polar ray paths, the standard deviation of the remaining 435 residuals to AK135 is 1.2 s.

2.3 Results

Absolute and differential traveltime residuals are displayed against the turning point longitude in Figs 1 and 2. The histograms in the bottom half of each figure show that observations of retrograde propagation mainly have ray turning points in the eastern hemisphere, while prograde propagation is predominantly associated with turning points in the western hemisphere. Hence, two interpretations are possible: either a velocity heterogeneity in the inner core leads to an apparent lack of reciprocity, or genuine non-reciprocity leads to an apparent velocity heterogeneity.

3 DISCUSSION

Genuine non-reciprocity would have to result from rapid differential motions, Lorentz forces on free electric charges or the Coriolis force of Earth rotation. Rapid differential motions in the core are unlikely and incompatible with the observed

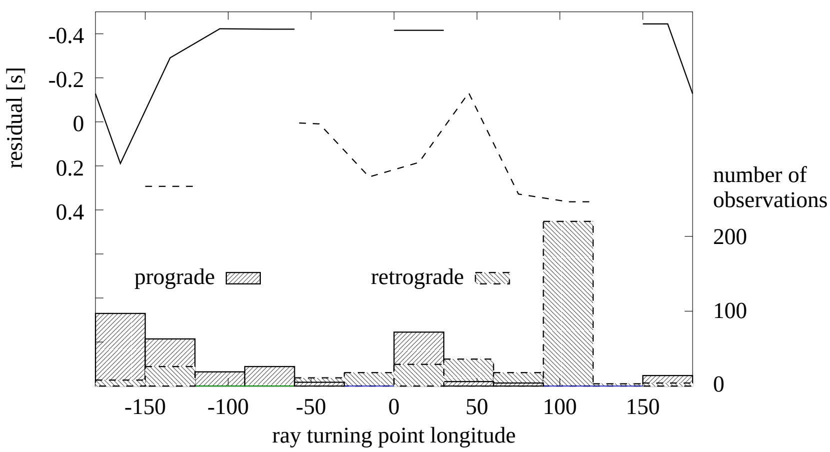

Figure 2. Same as Fig. 1, but for the combined data set of differential traveltimes. To facilitate comparison with Fig. 1, the *y*-axis is reversed, with positive residuals (fast inner core) pointing downwards. Again, only non-polar residuals are considered and bins with less than 10 observations are discarded.

even-degree splitting of free-oscillation multiplets (Widmer *et al.* 1992; Gilbert 1994). The core is probably a good conductor, arguing against the presence of high electric charge densities (Backus *et al.* 1996). Consistent with the observed effect, the Coriolis force of Earth rotation accelerates westward propagation. However, it fails to explain the observed traveltime residuals by several orders of magnitude. This is demonstrated in the following subsections.

3.1 Green tensor

Neglecting surface tractions, the receiver displacement at a location $\mathbf{x}_\mathrm{r}$ and time t can be written as

$$u_p(\mathbf{x}_\mathrm{r}, t) = \int_{V_\mathrm{s}} \int_{-\infty}^{t} F_q(\mathbf{x}_\mathrm{s}, \tau) G_p^q(\mathbf{x}_\mathrm{r}, \mathbf{x}_\mathrm{s}, t-\tau)\, d\tau\, dV_\mathrm{s}\,, \tag{1}$$

where the displacement $\mathbf{u} = (u_p,\ p = 1, \ldots, 3)$ is produced by the convolution of the body force density $\mathbf{F} = (F_q,\ q = 1, \ldots, 3)$ with the Green tensor $\boldsymbol{G}$, integrated over the source region V_s (Knopoff & Gangi 1959; Hudson 1980, p. 105). The component $G_p^q(\mathbf{x}_\mathrm{r}, \mathbf{x}_\mathrm{s}, t)$ of the system's Green tensor specifies the displacement response in the direction p at a location $\mathbf{x}_\mathrm{r}$ and time t due to an elementary point force at $\mathbf{x}_\mathrm{s}$ in the direction q at $t = 0$. Since the normal-mode eigenfunctions of the Earth's free oscillations constitute a complete basis for any deformation of the Earth, the Green tensor can be expressed as a sum over normal-mode displacements.

3.2 Non-rotating earth

On a non-rotating earth, the Green tensor can be written as (Dahlen & Tromp 1998, eq. 4.60)

$$G_p^q(\mathbf{x}_\mathrm{r}, \mathbf{x}_\mathrm{s}, t) = \sum_k \omega_k^{-1} s_k(\mathbf{x}_\mathrm{r})_p\, s_k(\mathbf{x}_\mathrm{s})_q \sin \omega_k t\,, \tag{2}$$

where k indexes the complete set of normal modes with eigenfrequency ω_k and eigenfunction $\mathbf{s}_\mathrm{k}$. These eigenfunctions are vector-valued, and $s_k(\mathbf{x}_\mathrm{r})_p$, for example, describes the displacement in the direction p at the receiver location. The reciprocity theorem follows directly from eq. (2):

$$G_p^q(\mathbf{x}_\mathrm{s}, \mathbf{x}_\mathrm{r}, t) = G_q^p(\mathbf{x}_\mathrm{r}, \mathbf{x}_\mathrm{s}, t)\,. \tag{3}$$

Consequently, a seismic wave generated at $\mathbf{x}_\mathrm{s}$ takes the same time to travel to $\mathbf{x}_\mathrm{r}$ as a wave from $\mathbf{x}_\mathrm{r}$ to $\mathbf{x}_\mathrm{s}$.

3.3 Effect of earth rotation

3.3.1 Non-reciprocity

With the Coriolis term $2\mathbf{v} \times \boldsymbol{\Omega}$ in the equation of motion, the normal-mode eigenfunctions become complex. Corresponding to eq. (2), the Green tensor of a rotating earth (Dahlen & Tromp 1998, eq. 4.127) can then be expressed as

$$G_p^q(\mathbf{x}_\mathrm{r}, \mathbf{x}_\mathrm{s}, t) = \mathscr{Re} \sum_k (i\omega_k)^{-1} s_k(\mathbf{x}_\mathrm{r})_p\, s_k(\mathbf{x}_\mathrm{s})_q^* \exp(i\omega_k t)\,. \tag{4}$$

Since $s_k(\mathbf{x}_\mathrm{r})_p\, s_k(\mathbf{x}_\mathrm{s})_q^*$ is generally not equal to $s_k(\mathbf{x}_\mathrm{s})_q\, s_k(\mathbf{x}_\mathrm{r})_p^*$, the Green tensor of a rotating earth does not satisfy the source–receiver reciprocity relationship given in eq. (3). Waves travelling in the direction of rotation (prograde) are slower than waves travelling against it (retrograde).

3.3.2 Coriolis splitting

The traveltime difference between westward and eastward propagation can be extrapolated from first-order Coriolis splitting of the corresponding free-oscillation modes. Any body wave can be represented as a sum of spheroidal normal-mode displacements ${}_n\mathbf{S}_\ell$, where n is the radial overtone number and ℓ is the degree of the mode. The oscillation frequency generally increases with n and ℓ so that body waves correspond to modes with high $n+\ell$. The deeper the penetration of a phase, the stronger the contribution of modes with $\ell \ll n$.

The receiver displacement ${}_n\mathbf{S}_\ell(\phi_\mathrm{r}, \theta_\mathrm{r})$ at longitude ϕ_r and colatitude θ_r due to an earthquake at $(\phi_\mathrm{s}, \theta_\mathrm{s})$ can be written as

$$\begin{aligned} {}_n\mathbf{S}_\ell(\phi_\mathrm{r}, \theta_\mathrm{r}) = \mathscr{L} P_\ell[&\cos\theta_\mathrm{r} \cos\theta_\mathrm{s} \\ &+ \sin\theta_\mathrm{r} \sin\theta_\mathrm{s} \cos(\phi_\mathrm{r} - \phi_\mathrm{s} + \beta_{n,\ell}\,\Omega\, t)] \sin\omega_0 t \end{aligned} \tag{5}$$

(Backus & Gilbert 1961), where $\mathscr{L}$ is a vector operator that depends on the type of source, $P_\ell(z)$ is the Legendre polynomial of degree ℓ, $\beta_{n,\ell}$ is the Coriolis splitting parameter (eq. 10 below) and ω_0 is the degenerate oscillation frequency of the mode on a non-rotating earth. From eq. (5) it follows that on a rotating earth the standing wave pattern of a spheroidal oscillation travels westwards at a speed of $\partial\phi/\partial t = \beta_{n,\ell}\,\Omega$.

3.3.3 Expected traveltime difference

For an estimate of the expected traveltime difference between eastward- and westward-propagating waves, consider eq. (5) for an earthquake on the equator at ($\phi_\mathrm{s} = 0^\circ$, $\theta_\mathrm{s} = 90^\circ$, $t = 0$). After the traveltime T, the displacement at an equatorial location ϕ_E to the east will be in phase with the displacement at $\phi_\mathrm{W} = -(\phi_\mathrm{E} + 2\beta_{n,\ell}\,\Omega\, T)$ to the west. This difference of $2\beta_{n,\ell}\,\Omega\, T$ in the epicentral distance translates into a traveltime difference

$$\delta T = 2\,\beta_{n,\ell}\,\Omega\, T\, \frac{\partial T}{\partial \phi}\,. \tag{6}$$

Together with the definition of the Coriolis splitting parameter $\beta_{n,\ell}$ (eq. 10 below), it follows that for a fixed overtone number n this time difference increases with decreasing ℓ, and hence with increasing penetration depth.

To estimate the magnitudes of these time differences for PKP_{DF} body waves, $\beta_{n,\ell}$ has to be evaluated for high frequencies. The principle of constructive interference on a sphere leads to Jeans' relation (Dahlen & Tromp 1998, p. 465):

$$\omega p = \sqrt{\ell(\ell+1)}\,, \tag{7}$$

relating the degree ℓ of a free-oscillation mode to the frequency ω and ray parameter p of the corresponding travelling wave. As suggested by Dahlen (personal communication, 1999), a simple estimate for $\beta_{n,\ell}$ follows from the asymptotic displacement eigenfunctions U and V of the PKP_{DF}-equivalent modes (Dahlen & Tromp 1998, p. 502),

$$U(r) \approx \frac{2\sqrt{q_\alpha}}{r\sqrt{\rho T}} \cos\left(\omega \int_{R_I}^{r} q_\alpha dr + \frac{\pi}{4}\right), \tag{8}$$

$$V(r) \approx \frac{2p}{r^2\sqrt{\rho T q_\alpha}} \sin\left(\omega \int_{R_I}^{r} q_\alpha dr + \frac{\pi}{4}\right), \tag{9}$$

with radial P-wave slowness q_α, density ρ, body wave traveltime T and turning point radius R_I. These approximations are valid for $R_I \ll r \leq a$, where a is the radius of the earth. The Coriolis splitting parameter for these displacement eigenfunctions is

$$\beta_n^\ell = \frac{1}{\ell(\ell+1)} \int_0^a (V^2 + 2\sqrt{\ell(\ell+1)}UV)\rho r^2 dr \tag{10}$$

(Dahlen & Tromp 1998, eq. 14.35). Evaluating eq. (10) with eqs (8) and (9) for high frequencies ω, the radial integral over the UV term becomes negligible due to the alternating sign of sin cos. The $\sin^2$ term in V^2 can be approximated by 1/2 for high ω. Furthermore, using Jean's relation (7) gives

$$\beta = \frac{2}{\omega^2 T} \int_0^a \frac{dr}{r^2 q_\alpha} \,. \tag{11}$$

Making use of the expression for the epicentral distance Θ (Dahlen & Tromp 1998, eq. 12.7),

$$\Theta = 2 \int_{R_I}^a \frac{p}{r^2 q_\alpha} dr \,, \tag{12}$$

we can simplify eq. (11) to

$$\beta \approx \frac{\Theta}{\omega^2 Tp} \,. \tag{13}$$

In this, it is assumed that contributions to β from displacements close to and below the turning point radius R_I are negligible. Together with eq. (6) and $p = \partial T / \partial \phi$, we obtain an estimate for the traveltime difference between equatorial eastward and westward PKP_{DF} propagation:

$$\delta T \approx \frac{2\Theta\Omega}{\omega^2} \,, \quad \text{where} \quad \Theta < 180^\circ \,. \tag{14}$$

For an epicentral distance of $\Theta = 160^\circ$ and $\omega = 1/s$, this amounts to 0.0004 s, which is three orders of magnitude smaller than the effect to be explained. Contrary to my previous assertion (Maus 1998), the observed traveltime residuals cannot be explained by the Coriolis force of earth rotation and must therefore be primarily due to velocity heterogeneities.

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