Scaling statistical analysis of magnetic and gravity data

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Summary

Statistical analysis is a powerful tool for the interpretation of potential field data. Recent findings on scaling properties of the density and susceptibility distribution in the Earth's crust have had a strong impact on statistical methods. I introduce the concept of scaling features of potential fields and show how scaling models can be used in the processing and interpretation of magnetic and gravity data for geological mapping and to determine depth.

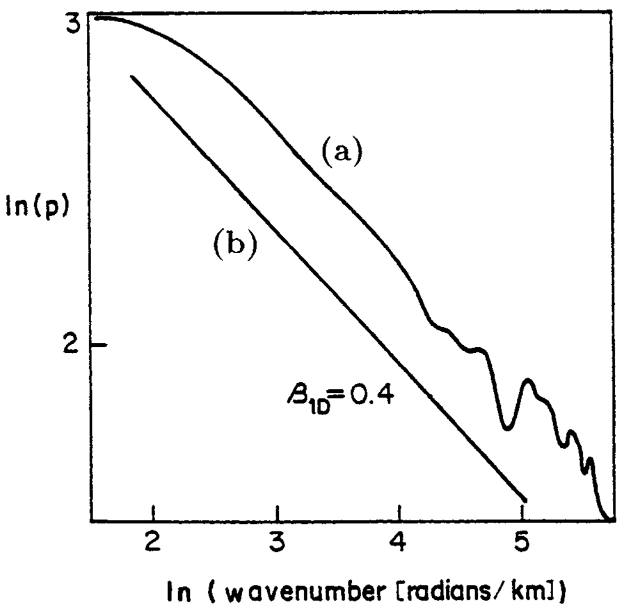

Figure 1: (a) Log-log power spectrum of the vertical susceptibility profile of the German Continental Deep Drilling Project pilot borehole. Blackman-Tukey estimate, smoothed by a Parzen window of 400-m maximum lag. The straight line (b) has a slope of -0.4 (after Maus & Dimri, 1995).

Power spectrum of the field caused by scaling sources

Statistical analysis of gravity and magnetic data requires realistic statistical models for the density and susceptibility distribution in the Earth's crust. Pilkington and Todoeschuck[1,2] showed that power spectra of density and susceptibility logs decay approximately linearly when plotted in log-log scale (Fig. 1)

$$\ln[P(k)] = -\beta \ln(k) + \ln(c)$$
$$\Rightarrow P(k) = c\ k^{-\beta}, \quad (1)$$

where k is the wavenumber and β and c are constants. Hence, these power spectra are proportional to a power of the wavenumber. The constant β is called a scaling exponent. Random functions with such a power spectrum are referred to as self-similar, scaling or fractal.

From a practical point of view, the scaling exponent reflects the ruggedness of a random function. The higher the scaling exponent the smoother the function. White noise corresponds to a scaling exponent of zero. When comparing scaling exponents, the dimension of the cross-section on which measurements were made has to be taken into account. The 1D scaling exponent of a susceptibility profile, for example, is approximately 1 less than the scaling exponent of a 2D susceptibility survey of the same area[3].

Scaling density and susceptibility distributions give rise to gravity and magnetic fields which in turn have scaling features[2-6] (Fig. 2). As in the case of the source distri-

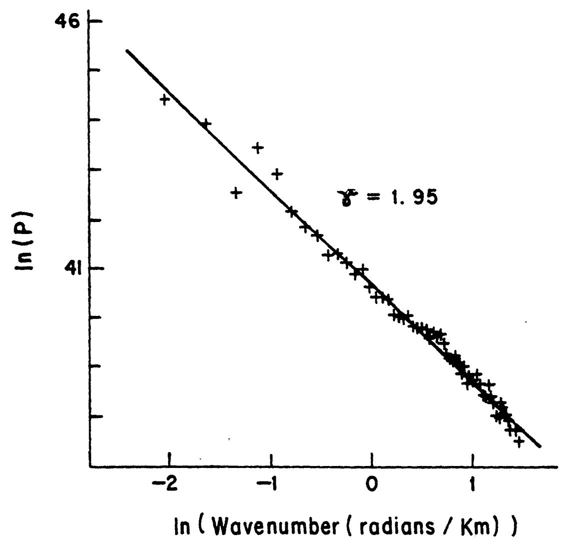

Figure 2: Radially averaged power spectrum of an aeromagnetic survey in the area of the German Continental Deep Drilling Project (KTB) after reduction to the pole and continuation downwards to ground level in log-log scale. The solid line has a slope of -1.95.

butions, there is a difference in the scaling exponents of 1D and 2D cross-sections of the field[3]. The theoretical 2D power spectrum $P(\vec{k})$ of the potential field in a horizontal observation plane caused by a half-space of scaling sources is given by

$$P(\vec{k}) = c_\gamma\ I\ \Theta(\vec{k})\ |\vec{k}|^{-\gamma}\ e^{-2z|\vec{k}|}, \quad (2)$$

where c_γ is a constant (for fixed values of γ), I is related to

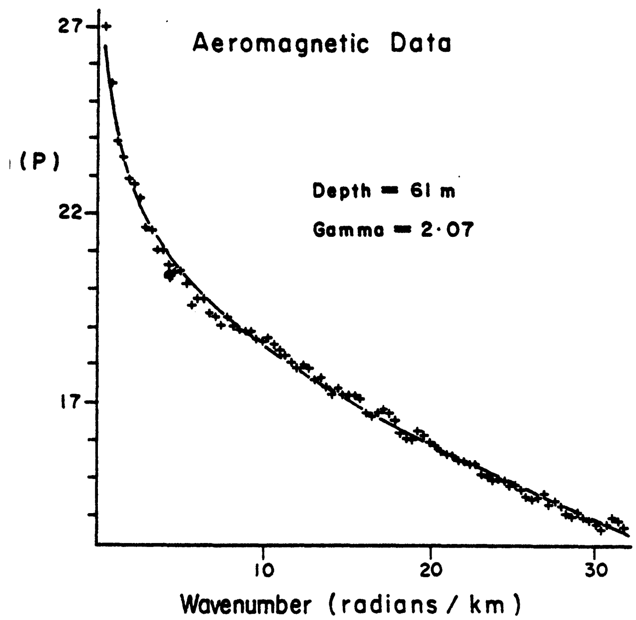

Figure 3: Radially averaged log power spectrum of the KTB aeromagnetic survey. The solid line indicates the model power spectrum for $\ln(I) = 19.23$, $z = 61m$, and $\gamma = 2.07$ (after Maus & Dimri, 1995).

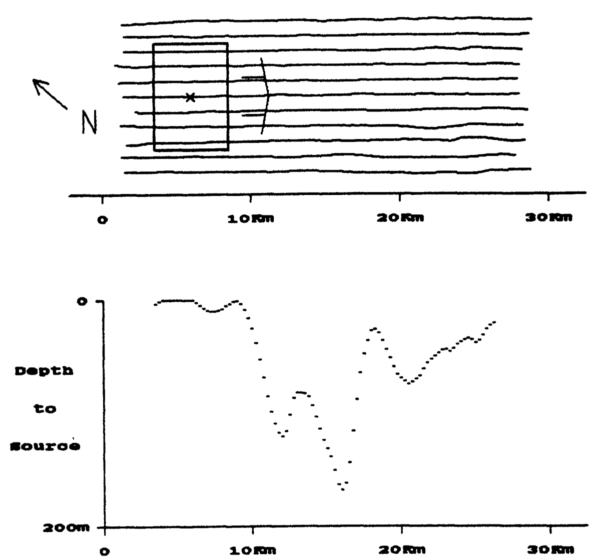

Figure 4: A window is moved over the survey area. For every position of the window an inversion is carried out using the data within the window. Here, the resulting depths to source are displayed as a profile.

the intensity of density or susceptibility variations, $\Theta(\vec{k})$ reflects the anisotropy of the magnetic field (for the gravity field $\Theta(\vec{k}) \equiv 1$), γ is the scaling exponent of the field and z is the distance between the observation plane and the top of the model half-space. Fig. 3 shows the fit of the model power spectrum defined by eq. (2) to the power spectrum of a real aeromagnetic data set. In this case the radially averaged power spectrum was used. However, radial averaging leads to a loss of information and future methods are likely to use the original 2D power spectrum, instead.

Applications

The model power spectrum defined by eq. (2) is governed by three parameters, z, I and γ, which can be derived by statistical analysis of a survey and can subsequently be displayed as maps. All three parameters describe different aspects of the gravity or magnetic field.

The depth z to the top of the sources is particularly interesting in the study of sedimentary basins, where it may reflect the depth to the crystalline basement or the depth to certain intra-basin geological structures.

The Intensity I quantifies the amplitude of susceptibility and density variations while taking into account the distance of the sources from the observation plane. Hence, the parameter I can be high even for a weak field anomaly, if the depth to the source anomaly is high at the same time. A further attractive feature of the parameter I is that it can be derived from magnetic data without a reduction to the pole.

The third parameter, the scaling exponent γ, reflects statistical properties of the source distributions. As discussed below, it may be useful in separating different types of geology, such as volcanic from metamorphic, or Recent from Precambrian.

The optimum values for these parameters are obtained by inversion. An appropriately sized ‘window is moved over the data set (Fig. 4), and an inversion of the data within the window is carried out to derive the best values for I, z and γ at every position of the window. The values for subsequent inversions (= window positions) can then be plotted as profiles (Fig. 4 & 5), or as maps of the particular parameter. However, due to a trade-off between the scaling exponent γ and the depth to source z, only one of these two parameters can be resolved at a time.

To obtain the depth to source, a constant scaling exponent has shown to be a reasonable assumption in areas without major changes in the geology of the basement. Pilkington and Todoeschuck[7] used a value of $\gamma = 3$, while 7 = 2 was used in another case study[8] (Fig. 4 & 5). Using a lower scaling exponent leads to greater depth estimates and vice-versa. Thus, the method can be calibrated by choosing the scaling exponent in such a way that the inversion yields the correct depth to source for a window with known basement depth.

If, on the other hand, the area has outcropping sources

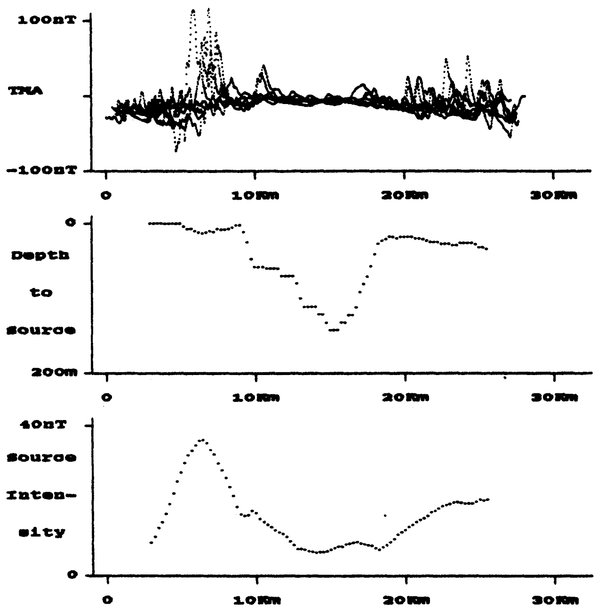

Figure 5: The magnetic profiles used for the inversion are plotted together in the top graph. The statistical method quantifies the ruggedness of these profiles in terms of the scaling exponent and the depth to source, whereas the amplitude of the field is reflected in the source intensity. For this particular study the scaling exponent was kept constant at $\gamma = 2$.

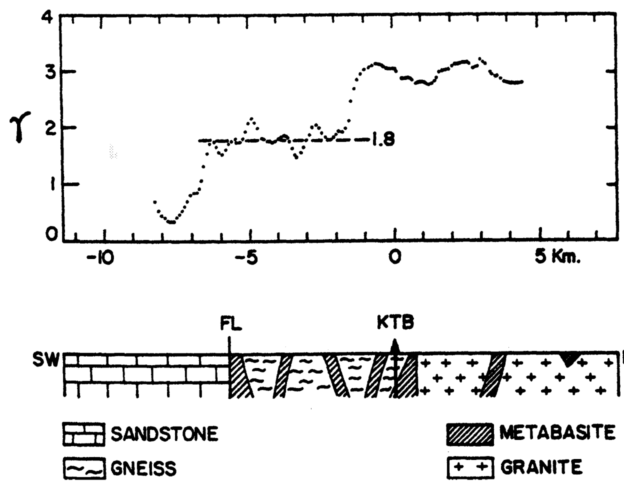

Figure 6: A SW-NE profile of the scaling exponent in the area of the German Continental Deep Drilling Project (KTB) with a cartoon of the geological section. The Franconian line, a major vertical fault, is indicated by FL (after Maus & Dimri, 1995).

then the depth to the top of the sources is known to be the survey terrain clearance. In this case one can derive maps of the Intensity I and the scaling exponent γ. Fig. 6 shows a profile of the scaling exponent over an area with outcropping magnetic rocks. It correlates well with the surface geology. The variations of the scaling exponent are thought to indicate differences in the distribution of magnetic minerals within the geological units".

Conclusions and outlook

Scaling source models have lead to a new understanding of the power spectrum of gravity and magnetic data. They have made it possible to map the topography of a crystalline basement covered by nonmagnetic sediments with increased precision'. As a by-product, a map of the intensity of magnetization of the basement can be obtained. For areas with outcropping sources, maps of the intensity and the scaling exponent of the source variations can be derived. Although this has so far only been tried for magnetic data[5], it may in fact be an attractive way of displaying gravity data.

Many of the theoretical questions regarding scaling features of potential fields are now quite well understood. Hence, the next step should be to develop efficient algorithms for high resolution mapping of the scaling parameters z, I and γ. Existing methods of spectral analysis can be improved by using an anisotropic 2D spectral model, instead of the radially averaged power spectrum. Furthermore, the loss of information during gridding can be avoided by estimating power spectra directly from the original point-located data.

References

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